

Chaos in an RLD Circuit

Ilana Balaskas, Eyüp Togay

*Department of Physics and Astronomy, University of Rochester**

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ABSTRACT

In this work, we investigate chaotic behavior in a driven RLD circuit. The diode acts as a nonlinear circuit element, and when the circuit is driven at large enough voltages, irregular oscillations characteristic of chaotic dynamics can emerge. Using time-domain measurements and phase-space diagrams, the circuit response reveals chaotic motion mixed with regions of regular, periodic behavior. Observing these transitions provides insight into the diode's nonlinear response and how it influences the overall circuit dynamics and oscillation period.

INTRODUCTION

Nonlinear electrical circuits provide a simple way for studying complex dynamic behaviors. In the case of this experiment, an RLD circuit driven by some external voltage, which as a result, exhibits time-dependent behavior. This makes the circuit a useful model for exploring the onset of chaos in physical systems. The RLD circuit consists of a resistor, an inductor, and a diode connected in series. The nonlinearity of the circuit is a result of the diode, which conducts only when the forward voltage across it exceeds some characteristic threshold V_D . As a result, the circuit alternates between two distinct states: reverse bias (non-conducting or "off") and forward bias (conducting or "on").

Chaos in the RLD circuit emerges both from the diode's nonlinear switching and its behavior in series with the inductor. The inductor introduces a memory effect; the current at the end of one driving cycle influences the initial conditions of the next. Therefore, the precise moment at which the diode switches between conducting and non-conducting states depends on the inductor current, which itself depends on the circuit's prior state. As a result, as the amplitude of the driving voltage is increased, these switching times shift in a nonlinear manner resulting in changes in the system's periodic response.

With increasing drive strength, the circuit undergoes a sequence of bifurcations, most notably period-doubling transitions that can be observed with the previously mentioned phase-space diagrams. We expect that as one continues to increase the driving voltage, the results will eventually yield an aperiodic behavior, or in the terms of our experiment, lead to some deterministic chaos. In this work, we investigate the onset of chaotic behavior in a driven RLD circuit and aim to illustrate how a sim-

ple nonlinear element can give rise to complex dynamical phenomena.

THEORY

Recall that in this circuit, the nonlinearity observed is a result of the diode switching between reverse and forward bias. When the diode is off, the circuit behaves as a simple RL circuit governed by

$$V_{\text{in}} = IR + L \frac{dI}{dt}. \quad (1)$$

Once the diode has reached its critical threshold voltage, it turns on, and there is now an additional term such that,

$$V_{\text{in}} = IR + L \frac{dI}{dt} + V_D. \quad (2)$$

During this conducting phase, the diode conducts when there is a positive voltage across it, such that

$$V_d = V_b = -V_f, \quad (3)$$

where V_f is the forward voltage of the diode. When the diode is reverse biased and not conducting, it behaves as a capacitor with capacitance C . If the voltage across the diode is time-dependent, some current flows even when there is no longer conduction occurring.

When the diode is conducting, the (negative) current in the circuit is given by

$$I(t; A) = \frac{V_0}{\sqrt{R^2 + L^2\omega^2}} \cos(\omega t - \theta_a) + Ae^{-Rt/L} + \frac{V_f}{R}, \quad (4)$$

where V_0 and ω are the amplitude and angular frequency of the driving voltage, and R and L are the resistance and inductance of the circuit. A is a constant determined by the initial conditions which represents the current decaying out of the inductor. The phase angle in the conducting stage is given by

$$\theta_a = \arctan\left(\frac{\omega L}{R}\right). \quad (5)$$

When the diode is no longer conducting, it behaves as a capacitor and the current takes the form of

$$I(t; B, \Phi) = \frac{V_0}{\sqrt{R^2 + \frac{L^2}{\omega^2}(\omega^2 - \omega_0^2)}} \cos(\omega t - \theta_b) + Be^{-(R/2L)t} \cos(\omega_b t + \Phi) \quad (6)$$

where B and Φ are constants. The natural frequency of the circuit in this stage is

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad (7)$$

and the damped oscillation frequency is

$$\omega_b^2 = \omega_0^2 - \left(\frac{R}{2L}\right)^2. \quad (8)$$

The phase angle in this regime is

$$\theta_b = \arctan\left(\frac{L(\omega^2 - \omega_0^2)}{R\omega}\right). \quad (9)$$

The diode voltage in the non-conducting stage is

$$V_D = V_b = V_0 \cos(\omega t) - I(t; B, \Phi)R - L \frac{dI(t; B, \Phi)}{dt}. \quad (10)$$

While an ideal diode stops conducting when the current reaches zero, a real diode, such as the one in this circuit, continues to conduct for some finite recovery time τ_r , such that

$$\tau_r = \tau_m \left(1 - e^{-|I_m|/I_c}\right), \quad (11)$$

where τ_m and I_c are constants, and I_m is the maximum forward current in the previous cycle. This recovery time causes the previously mentioned memory effect; disregarding the first cycle, the behavior of the diode in a given phase is dependent on the current in the previous phase.

The cycles of the circuit are characterized by that maximum forward current, I_m . Since I_m changes every second, fourth, etc cycle, a one-dimensional map is formed where the effect from cycle to cycle can be better understood. This map depends on the external control parameter

$$\lambda = \frac{V_0}{V_f}. \quad (12)$$

As λ varies, the system exhibits bifurcations and period doubling that eventually lead to chaotic behavior.

There is some quality factor Q , defined as

$$Q = \omega \frac{\text{energy stored}}{\text{average power dissipated}}. \quad (13)$$

where $w = 2\pi v$ The quality factor determines the response of the system to the external driving force.

For our RLC circuit, the magnitude of the maximum frequency, which is slightly less than the resonance frequency value, has some quality factor, defined is

$$Q = \frac{\omega_0 L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}. \quad (14)$$

EXPERIMENTAL SETUP

Circuit set-up is a fairly simple implementation of the schematic below. The function generator providing the driving voltage for the circuit is connected to a resistor measured to be $120k\Omega$. The resistor is then set in series with the inductor ($10mH$) and diode (1N4007), which are configured in series in a remade box with BNC ports on each side (Note: the diode in the box is flipped to schematic. To fix, simply flip the box and connect it to the circuit "backwards"). With the one BNC port connected to the resistor, the other side is connected to a Picoscope 4262 which is wired to the computer for data collection. Below is a simple schematic of the circuit.

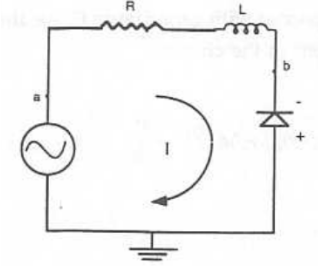


FIG. 1. Schematic of the experimental setup.

Before collecting data, one must first find the resonant frequency for the circuit. To do so, begin by setting the function generator $V_o = 20mV$. This is low enough that the diode is not in saturation mode. In the triode region, one can use the driving frequency to properly maximize the diode voltage. One can do so by slowly increasing the driving frequency and observing on the oscilloscope where a maximum is met for the diode voltage. Per the manual, it is recommended to feel the resonant frequency at $100kHz$. But, it was found that for our circuit a resonant frequency of $205kHz$ was sufficient to still properly complete data collection.

Now, with the driving frequency set to our found resonant frequency $V_o = 205kHz$, one can now work to observe the earlier theorized period doubling and chaos by slowly increasing the driving voltage and monitoring on the oscilloscope. Up to four bifurcations can be observed with this set-up. We additionally hope to observe "islands of stability" in the chaotic regime as we increase our driving voltage. Once data is collected, conclusions can be drawn regarding the chaotic behavior of the circuit and one can also calculate the forward-biased voltage of the diode, V_f .

RESULTS

The voltage across the diode, V_d , and the current through the circuit were measured simultaneously. From the resulting current-voltage characteristic, the region where the diode begins conducting was identified, and the corresponding voltage drop is taken as the forward bias voltage, V_f . When diode is non-conducting (off), the system behaves like a linear RL circuit. We can then increase the driving amplitude until we notice the diode is turning on. Doing so resulted in a measured forward-biased voltage of 0.8V. This is consistent with the datasheet for the 1N4007 diode.

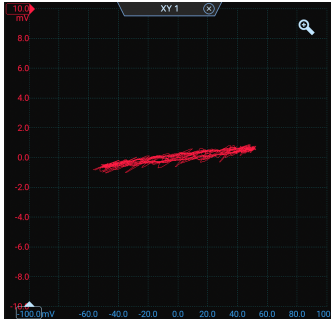


FIG. 2. *Moment diode was observed to be conducting.*

Following the experimental procedure, a number of graphs were collected at various voltage for where period doubling and chaos was observed. Plots of Driving Voltage (x) vs Output Voltage (y) were configured for Period-1, Period-2, and Period-4 behavior. Below is each graph and their corresponding driving voltages where behavior was observed.



FIG. 3. *Period-1 Behavior observed at 1.5V.*

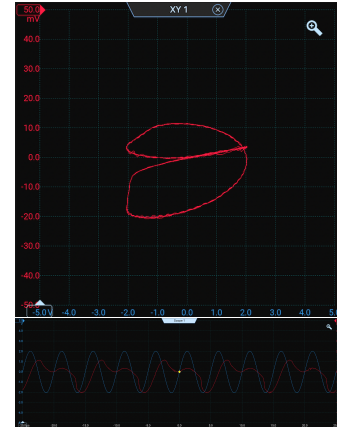


FIG. 4. *Period-2 Behavior observed at 4.2V.*

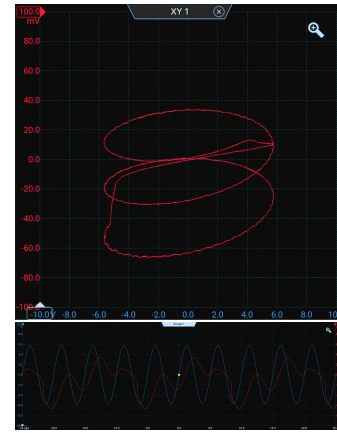


FIG. 5. *Period-4 Behavior observed at 11.7V.*

After increasing the driving voltage further, chaos was finally observed at 20V. One can get a better understanding of the period-doubling by comparing the time-domain waveforms of each behavior.

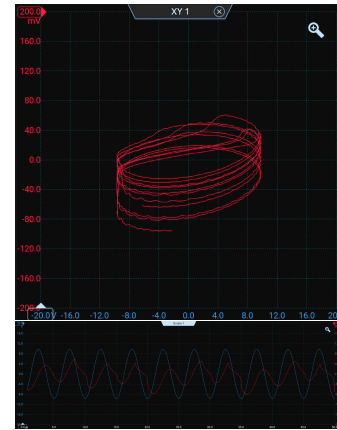


FIG. 6. *Chaotic Behavior observed at 20.0V.*

Further, as expected, an "island of stability" was ob-

served from 11.6V to 11.7V. An island of stability is a region of stable, predictable behavior in an otherwise chaotic region. Below are the plots for comparison.

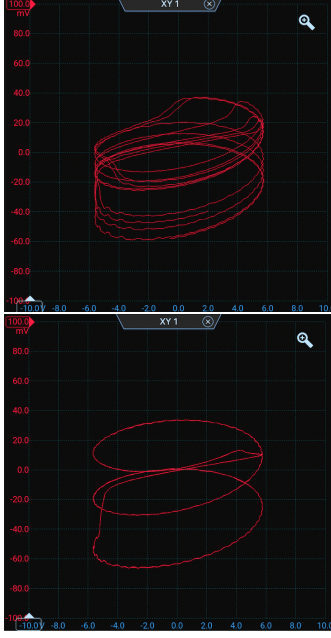


FIG. 7. *Island of stability observed from 11.6V to 11.7V.*

DISCUSSION

The experimental results show that while an RLD circuit is very simple to construct and implement, it exhibits a large range of nonlinear dynamical behavior. It is found that as the driving voltage amplitude is increased, the circuit transitions through a clear sequence of periodic responses before finally entering a chaotic regime. This progression is consistent with the expected period-doubling route to chaos described in the theoretical model.

The phase-space diagrams provided a particularly clear visualization of these transitions between various behaviors. Periodic behavior produces closed, well-defined loops, while period-doubled states result in multiple distinct loops corresponding to the increased oscillation period. When chaotic regions are observed, the phase-space plots are much less defined. In these instances, there are no longer simple closed curves, but rather the nonlinear dynamics that point toward chaos. These observations align with the one-dimensional iterative map framework discussed in the theory section. We expect that as the maximum forward current produced in one cycle will determine the behavior in subsequent cycles, resulting in the altered behavior observed.

The occurrence of nonlinear behavior is due primarily to the diode. As the diode switches between conducting and non-conducting states, an alternating response

is introduced into the circuit. While the resistor and inductor alone would yield linear dynamics (which is what is observed when the diode is off, and therefore not in saturation mode), the diode's forward voltage threshold and recovery time cause the circuit's behavior to change within each driving cycle.

While the diode plays a key role in this nonlinear behavior, we must also credit the inductor for its contribution in forming the memory effect. Recall, for this circuit, the current at the end of one cycle influences the initial conditions of the next. Together, these effects result in a sensitive dependence of altering behavior as driving amplitude is varied.

This dependence is observed as relatively small changes in the driving voltage which led to qualitatively different behavior being observed. Particularly, the transition from periodic to chaotic motion was observed over a very small range of driving voltages, and in special cases, it is observed in the earlier mentioned islands of instability. These islands correspond to temporary regions of periodic stability within the chaotic regime, and the presence of these islands supports the conclusion that the irregular motion observed is deterministic chaos as opposed to random noise being observed on the oscilloscope.

EXPERIMENTAL LIMITATIONS

Several experimental limitations may have influenced the precise values of the voltages at which bifurcations were observed. There is a level of component tolerances, particularly in the diode and inductor. While we found a resonant frequency of 205kHz to be an acceptable value, there is, of course, error to be considered in how that may have affected the signal analyzer's ability to monitor the frequencies as the driving voltage was varied. There are also possible shifts in the forward-biased voltage and recovery time that were not accounted for that could have had an effect on data collection. Most notably, we must take into consideration how measurement noise and finite oscilloscope resolution may have affected the graphs, especially at points where changes in behavior were marked.

CONCLUSION

Through a driven RLD circuit, a range of nonlinear dynamical behavior was observed at various driving voltage. The circuit transitioned from periodic motion to period-doubled responses and ultimately to chaotic behavior following the Period-4 regime. These transitions in behavior were a result of the diode's nonlinear switching and recovery time, along with the memory effect caused by the inductor. Furthermore, islands of stability were observed within the chaotic regime, consistent with deterministic

chaos rather than random noise. Overall, this experiment demonstrates how a simple nonlinear circuit can exhibit complex dynamical behavior through a well-defined bifurcation processes.

* PHYS 243W, Instructor: Professor Nicholas Bigelow

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